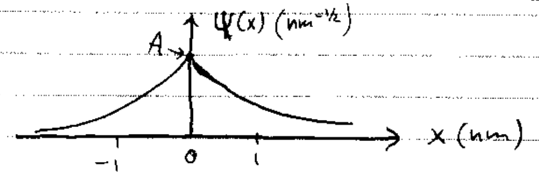
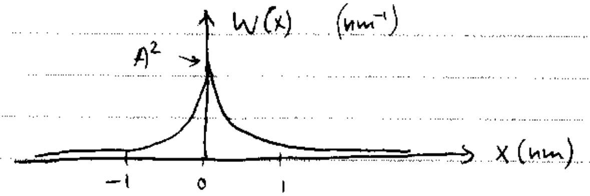
Antwoorden Midterm toets Kwantum fysica 1

10 maart 2006

Problem 1

a)





b) Normalized if $\int_{-\infty}^{\infty} |\Psi(x)|^2\, dx = 1$

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = A^2 \int_{-\infty}^{\infty} e^{-2\left|\frac{x}{b}\right|} dx = 2A^2 \int_0^{\infty} e^{-2\frac{x}{b}} dx = \frac{2A^2 b}{-2}\left[e^{-2\frac{x}{b}}\right]_0^{\infty} =$$

$$\frac{2A^2 b}{-2}[0-1] = A^2 b \quad \Rightarrow \text{ This is } 1 \text{ for } A = 1\ nm^{-1/2}$$

$|\Psi(x)|^2 = W(x)$ is a probability density, so $W(x)dx$ is a dimensionless number, so A must be of dimension $\frac{1}{\sqrt{\text{length}}}$

c) $\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \Psi^*(x)\, \hat{x}\, \Psi(x)\, dx = \int_{-\infty}^{\infty} A e^{-\left|\frac{x}{b}\right|} x A e^{-\left|\frac{x}{b}\right|} dx =$

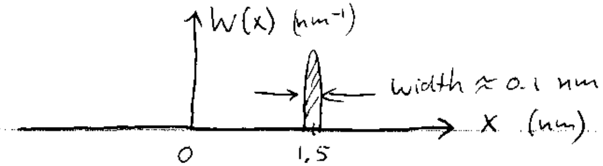
$$A^2\left(\int_{-\infty}^{0} x e^{+\frac{2x}{b}} dx + \int_0^{\infty} x e^{-\frac{2x}{b}} dx\right) = A^2\left(\int_0^{\infty} -x' e^{-\frac{2x'}{b}} dx' + \int_0^{\infty} x e^{-\frac{2x}{b}} dx\right) = 0$$

d) $\Psi(x)$ is symmetric around $x=0$, so $\langle \hat{x} \rangle$ must be 0.

e) $P(0\,nm < x < 1\,nm) = P_{01} = \int_0^{1\,nm} W(x)\, dx = \int_0^{1\,nm} A^2 e^{-2\frac{x}{b}} dx =$

$$\frac{A^2 b}{-2}\left[e^{-2\frac{x}{b}}\right]_0^{1\,nm} = -\frac{1}{2}\left[e^{-2\frac{1}{1}} - 1\right] = \frac{1-e^{-2}}{2} \approx 0.43$$

f)



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$$\int_{-\infty}^{\infty} W(x)dx = 1 \Rightarrow \text{area} = 1 \Rightarrow \text{height is } \frac{1}{0.1\,nm}$$

Problem 2

a) Show that $\langle \Psi_s | \Psi_s \rangle = 1$

Use that $\langle \varphi_m | \varphi_n \rangle = 1$ for $m=n$, and 0 for $m \neq n$.

$$\langle \Psi_s|\Psi_s\rangle = |c_1|^2\langle\varphi_1|\varphi_1\rangle + |c_2|^2\langle\varphi_2|\varphi_2\rangle + |c_3|^2\langle\varphi_3|\varphi_3\rangle$$
$$= \frac{4}{9} + \frac{3}{9} + \frac{2}{9} = 1$$

b) $\langle \hat{A} \rangle = \langle \Psi_s | \hat{A} | \Psi_s \rangle = |c_1|^2\langle\varphi_1|\hat{A}|\varphi_1\rangle + |c_2|^2\langle\varphi_2|\hat{A}|\varphi_2\rangle + |c_3|^2\langle\varphi_3|\hat{A}|\varphi_3\rangle$
$+ 2c_1c_2\langle\varphi_1|\hat{A}|\varphi_2\rangle + 2c_1c_3\langle\varphi_1|A|\varphi_3\rangle + 2c_2c_3\langle\varphi_2|\hat{A}|\varphi_3\rangle$

Here $c_1^*c_2 + c_2^*c_1 = 2c_1c_2$

$$= \frac{4}{9}\cdot -A_0 + \frac{3}{9}A_0 + \frac{2}{9}\cdot 2A_0 + \left(\frac{4\sqrt{3}}{9} + \frac{4\sqrt{2}}{9} + \frac{2\sqrt{6}}{9}\right)4A_0 = \left(\frac{1}{3} + \frac{4\sqrt{3}+4\sqrt{2}+2\sqrt{6}}{9}\right)A_0$$

c) $|\Psi(t)\rangle = \hat{U}|\Psi_0\rangle = e^{-\frac{i\hat{H}t}{\hbar}}|\Psi_0\rangle$ and $\langle\Psi(t)| = \langle\Psi_0|\hat{U}^\dagger = \langle\Psi_0|e^{+\frac{i\hat{H}t}{\hbar}}$

$$\langle\hat{A}\rangle(t) = \langle\Psi(t)|\hat{A}|\Psi(t)\rangle = \langle\Psi_0|\hat{U}^\dagger\hat{A}\hat{U}|\Psi_0\rangle$$

$$= \left(c_1^* e^{+\frac{iE_1t}{\hbar}}\langle\varphi_1| + c_3^* e^{+\frac{iE_3t}{\hbar}}\langle\varphi_3|\right)\hat{A}\left(c_1 e^{-\frac{iE_1t}{\hbar}}|\varphi_1\rangle + c_3 e^{-\frac{iE_3t}{\hbar}}|\varphi_3\rangle\right)$$

$$= c_1^2\langle\varphi_1|\hat{A}|\varphi_1\rangle + c_3^2\langle\varphi_3|\hat{A}|\varphi_3\rangle + c_1c_3\left(e^{i\frac{E_3-E_1}{\hbar}t}\langle\varphi_3|\hat{A}|\varphi_1\rangle + e^{i\frac{E_1-E_3}{\hbar}t}\langle\varphi_1|\hat{A}|\varphi_3\rangle\right) =$$

$$= \frac{-4}{9}A_0 + \frac{10}{9}A_0 + 2\cdot\frac{2\sqrt{5}}{9}\cos\left(\frac{E_3-E_1}{\hbar}\cdot t\right)4A_0 =$$

$$\frac{6}{9}A_0 + \frac{16\sqrt{5}}{9}\cos\left(\frac{E_3-E_1}{\hbar}\cdot t\right)A_0$$

Amplitude is $\frac{16\sqrt{5}}{9}A_0$ Only frequency is $f = \frac{E_3-E_1}{2\pi\hbar}$

$\omega = 2\pi f$